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Where is the Luttinger liquid in one dimensional semiconductor quantum wire structures?

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We present the theoretical basis for analyzing resonant Raman scattering experiments in one-dimensional systems described by the Luttinger liquid fixed point. We make experimentally testable predictions for distinguishing Luttinger liquids from Fermi liquid and argue that presently available quantum wire systems are *not* in the regime where Luttinger liquid effects are important.

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It is theoretically well-established [1-3] that a one dimensional interacting electron system (1DES), is *not* a Fermi liquid (FL). Unlike a Fermi liquid, the interacting 1DES has neither sharp fermionic quasiparticle excitations nor a discontinuity in the electron momentum distribution function. The elementary excitations are charge e, spin zero bosons and spin $1/2$ charge 0 'semions' (fractional statistics objects), and the fermion is a composite of these. Interacting 1DES have been generically termed Luttinger Liquids (LL) [2] and have been the subject of extensive theoretical study over the last 40 years and particularly over the last decade. Despite the intense theoretical interest, there have been few convincing experimental demonstrations of the predicted LL behavior in real 1DES. The power-law density of states observed in tunneling into edges of quantized hall systems [4] have been interpreted in terms of the theoretically expected 'chiral Luttinger liquid' behavior of edge states. The origin of the differences between the observed and expected exponents is presently an area of active inquiry. Photoemission experiments on Mott insulating oxides have been interpreted in terms of the 'holon' and 'spinon' excitations of a charged Luttinger liquid [5].

A 1DES which is of particular interest both for fundamental physics and for technology is the system formed in GaAs-based semiconductor quantum wire (QWR) structures. Modern materials growth and fabrication techniques have produced nearly ideal 1DES in which the electron may move freely only along the length of the wire. The transverse motion is quantized with the quantum 1D subbands separated by several meV. It is possible to have low enough carrier densities so that at low temperatures only the lowest 1D subband is occupied by electrons. Such GaAs QWR based 1DES should be ideal systems for the study of interacting electrons in one dimension because they are free from complications arising from band structure, lattice effects, and crossovers to three dimensional behavior which often make interpretations of experimental data difficult in more traditional 1DES based on organic compounds.

It is surprising, therefore, that no definitive LL behavior has been reported in GaAs QWR systems, and in fact the 1D Fermi gas/liquid model seems to "work" operationally very well in describing and explaining the observed 1DES experimental properties in GaAs QWR [6,7]. Part of the reason for the apparent absence of the expected LL behavior is undoubtedly the fact that in weakly interacting 1DES, *at finite temperatures and in the presence of impurity scattering*, the actual quantitative difference between a LL and a FL is not large [7], although the qualitative conceptual difference between the two is huge. A more fundamental issue is that the differences between a Luttinger Liquid and a Fermi liquid are most obvious in the one-electron spectrum, while the experimental probes which may most conveniently be applied to the QWR structures produce 'particle-hole pairs'. The differences in particle-hole pair properties between Luttinger Liquid and Fermi Liquid systems is much less pronounced than are the differences in the one electron spectrum. This perhaps accounts for the fact that one of the most important probes of QWR structures, resonant inelastic light scattering or Raman scattering spectroscopy (RRS) [6,8], has not yet observed any definitive indications of LL behavior in these systems.

In RRS experiments, light is absorbed at one frequency and re-emitted at another, creating one or more particle-hole pairs. In the so-called polarized geometry with the incident and outgoing photons having the same polarization (so that no spin is transferred to the QWR), RRS experiments in GaAs QWRs consistently [6,8] show two peaks which indeed look qualitatively very similar [9] to the spectra for the corresponding 2D and 3D systems. In these higher dimensional systems, the two peaks have a clear and generally accepted Fermi liquid interpretation [9]. The higher energy peak is associated with the plasmon or charge density excitation (CDE), a collective density excitation of the electron gas, and the lower energy spectral peak is associated with incoherent particle-hole pair excitations (SPE). In the QWR materials, the lower energy peak occurs at an (approximate) excitation energy of $\sim qv_F$, where q is the excitation momentum and v_F is the 1D Fermi velocity obtained from the band

structure of the QWR. An interpretation of the lower peak as an SPE contribution seems therefore natural [9]. However, there is a strong theoretical objection to this interpretation: in a one dimensional system there is spin-charge separation: the only charge excitations live at the plasmon frequency, and cannot contribute to excitations at the SPE energy. The signal observed in this q, ν range must be due to the chargeless spin excitations of the LL; in particular it is possible to combine two $S = 1/2$ excitations into a $S = 0$ object, creation of which is allowed by the Raman selection rules.

Sassetti and Kramer (S-K) presented a qualitative theory of this effect [10]. They showed that although the leading contribution to the RRS matrix element corresponds to coupling the light to the electron density operator, there is a sub-leading term (which becomes more important under resonance conditions) which may be interpreted as a coupling of light to the energy density fluctuations of the electrons in the QWR. The energy density fluctuations have a contribution from the spin excitations, which qualitatively explains the data, but the S-K theory did not calculate the spectral weights of the RRS peaks. Too close to resonance, the S-K theory breaks down. The S-K work also does not show how to distinguish a LL from a FL in the RRS experiment. The most important theoretical problem is that the S-K calculation is logically inconsistent, because it uses an expression for the RRS matrix element which is correct only if the conduction band is a FL not an LL. Thus S-K uses FL matrix elements but LL excitations. In our paper the correct LL matrix element is used, leading to expressions different from those derived by S-K.

In this paper we present an essentially complete treatment of RRS in a one dimensional electron gas. We obtain a precise expression for the energy transferred to the QWR in a RRS experiment, valid at all values of the difference of the energy from resonance, and evaluate it quantitatively in several experimentally relevant limits. We show which features of the data contain information about the LL exponents, obtain expressions for the relative amplitudes of the SPE and CDE peaks, determine lineshapes and discuss qualitatively the crossover from LL to FL behavior.

Resonant Raman scattering is a two-photon process in which a photon is absorbed, transferring an electron from the valence (V) band to the conduction (c) band and a photon is emitted, transferring an electron from the conduction band back to the valence band. We assume that the valence band is initially filled, and assume there is no excitonic interaction between conduction and valence band states. The excited valence hole is then described by a single-particle Hamiltonian, which we write as H_V, while the conduction band is described by some interacting Hamiltonian which we denote H_{LL}. We denote the photon absorption and emission by $P_{1,2}$ respectively. The RRS process is described by the following Hamiltonian:

$$H = H_V + H_{LL} + \widehat{P}_1 + \widehat{P}_2 \tag{1}$$

where the photon-in (P_1) and photon-out (P_2) terms are

$$\widehat{P}_1 = e^{-i(\Omega+\nu/2)t} \sum_{p,s} c^\dagger_{p+q/2,s}(t) v_{p,s}(t) + \text{h.c.} \tag{2}$$

$$\widehat{P}_2 = e^{i(\Omega-\nu/2)t} \sum_{p,s} v^\dagger_{p,s}(t) c_{p-q/2,s}(t) + \text{h.c.} \tag{3}$$

with c and v the annihilation operators for electrons in conduction and valence band states respectively. Note that the operator $v^\dagger_{p,\sigma}$ creates an eigenstate of H_V with energy E^V_p while the $c^\dagger_{p,\sigma}$ operators does *not* create eigenstates of H_{LL}. The absorbed(emitted) photon energy and momentum are set $\Omega \pm \nu/2$ and $\pm q/2$ respectively.

We now use the standard methods of time-dependent perturbation theory to calculate the amplitude, $a_n(t_0)$, for the system at some time t_0 to be in some excited state $|n\rangle$ of QWR, but with no holes in the valence band. We assume the system is in its ground state at $t = 0$. Our neglect of any excitonic interaction between conduction and valence band simplifies the calculation and we obtain

$$a_n(t_0) = \frac{1}{L} \sum_{r,s} \int dR e^{-iqR} \int_0^{t_0} dT e^{i\nu T} \langle n | \widehat{O}_{rs}(R,T) | 0 \rangle \tag{4}$$

with

$$\widehat{O}_{rs}(R,T) = \int dx \int_0^T dt\, \phi(x,t) \times \psi_{rs}(R + x/2, T + t/2) \psi^\dagger_{rs}(R - x/2, T - t/2), \tag{5}$$

where r and s are band and spin indices ($pm1$), and

$$\phi(x,t) = e^{i\Omega t} \sum_p e^{i(E^V_p t - px)}. \tag{6}$$

Eqs. (4) and (5) are our fundamental new results: they show that the RRS process acts to create a particle-hole pair at a spatial separation x and temporal separation t. These are determined by the average photon frequency Ω and the valence-band properties encoded in E^V_p. Further, if interactions are present in the conduction band, the states created by $\psi^\dagger$ and by ψ are not eigenstates of H_{LL} and therefore the matrix element is itself modified by interactions.

We note that Eqs. (4) and (5) maybe substantially simplified in the limit of greatest physical interest. We linearize the valence band energy about the conduction band Fermi momentum, writing $E^V_F = -\Delta - v^V_F(rp - p_F)$ for branch r and define $\omega_R = \Omega - \Delta$ as the photon frequency with respect to the resonance energy, Δ. The $p-$integral gives $\delta(x + v^V_F t)$. Finally we write the conduction band operators in terms of the bosons which create eigenstates of H_{LL}, and normal-order in the boson basis, obtaining

$$\widehat{O}_{rs}(R,T) = L \int_0^T dt\, e^{i\omega_R t} G^c_{rs}(-r v^V_F t, t) : e^{i\Phi_{rs,\rho}(R, -r v^V_F t; T, t)} :: e^{i\Phi_{rs,\sigma}(R, -r v^V_F t; T, t)} :, \tag{7}$$

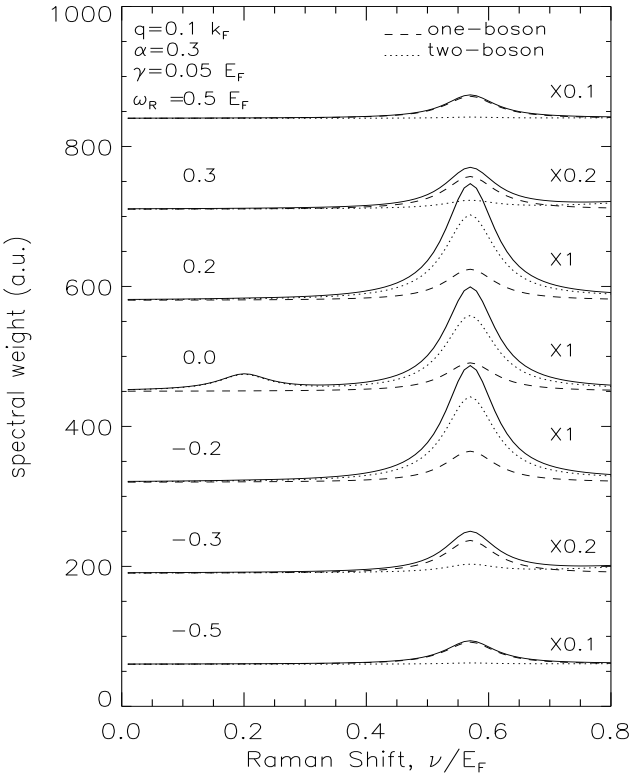

FIG. 1. Calculated polarized RRS spectra for various resonance condition, ω_R. One- and two-boson contributions have been plotted separately in order to show their relative contributions (see text). A finite broadening γ has been used to depict the results. Note that the overall spectral weights decreases dramatically off-resonance, as indicated by the individual scale factors on right side of each plot.

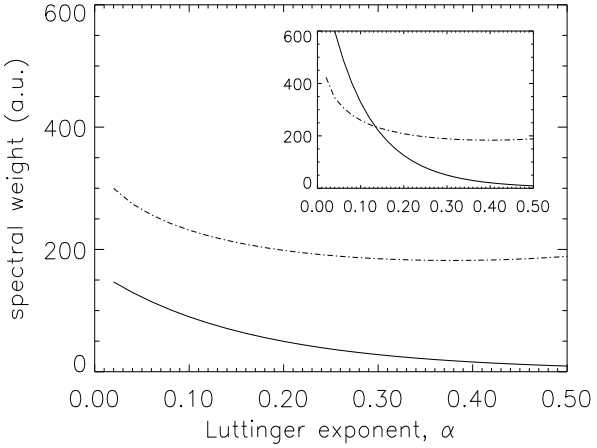

FIG. 2. Spectral weights for the low energy (solid curve) and the high energy (dashed) RRS peaks in the LL theory plotted A function of the Luttinger exponent α: $\omega_R = 0$ (main); 0.1 (inset). When $|\omega_R|$ larger than 0.1 the low energy ("SPE") weights are always much smaller than the high energy (CDE) weights over the whole range of α.

where

$$\Phi_{rs,\rho}(R,x;T,t) = 2\sum_{p>0} e^{-\alpha p/2}\sqrt{\frac{\pi}{pL}}\times$$
$$\Big\{-\sinh\theta_\rho \sin[p(rx+v_\rho t)/2][b^\dagger_{-rp}e^{ip(rR+v_\rho T)}+\text{h.c.}]$$
$$+\cosh\theta_\rho \sin[p(rx-v_\rho t)/2][b^\dagger_{rp}e^{-ip(rR-v_\rho T)}+\text{h.c.}]\Big\}, \quad (8)$$

$$\Phi_{rs,\sigma}(R,x;T,t) = 2s\sum_{p>0} e^{-\alpha p/2}\sqrt{\frac{\pi}{pL}}\times$$
$$\Big\{\sin[p(rx-v_F^c t)/2][\sigma^\dagger_{rp}e^{-ip(rR-v_F^c T)}+\text{h.c.}]\Big\}. \quad (9)$$

Here b^+ and σ^+ create charge and spin excitations respectively and $v_\rho = v_F^c e^{-2\theta_\rho}$ is the plasmon velocity, where the exponent $e^{-2\theta_\rho} = \sqrt{1+2g/\pi v_F^c}$ is defined for the short-ranged interaction, g. G^c is the exact conduction band Green's function at spatial separation $-rv_F^V t$, and temporal separation t. We have assumed that the interactions are negligible in the spin sector and therefore the spin excitation velocity is just the Fermi velocity. As long as v_F^V, the valence band velocity at the conduction band p_F is different from the spin and charge velocities of Luttinger liquid, G^c is a decaying function of t. In the noninteracting case, $G^c \sim 1/t$; interaction leads to a faster decay: $G^c \sim 1/t^{1+\alpha}$ with the LL exponent $\alpha = \sinh^2\theta_\rho > 0$ (not the same one as we use in Eqs. (8-9) for infinitely small convergent factor) for short-ranged interactions; G^c decays faster with the physically relevant long-ranged interactions. This faster decay of G^c is the mathematical expression of the renormalization of the RRS vertex by the interactions, which produce the Luttinger liquid behavior. As we will now show, it has important consequences for various aspects of the RRS spectra; and in particular for the dependence of the CDE and SPE energies on the difference of the average photon energy from the resonance.

We defer to a subsequent paper a full evaluation of the RRS correlation function, which is computationally demanding and not very illuminating, and present here the results of expanding Eq. (7) in terms of boson operators. The essential point is that if the combination of $e^{i\omega_R t}G^c(-rv_F^v t,t)$ decays rapidly as t increases (large ω_R as off-resonance or large α as strong interaction), then the $t-$integral is dominated by small times and an expression in power of bosons is rapidly convergent. We will show below that the first order term, one-boson result, gives the main contribution to CDE spectrum and dominates the whole spectrum as off-resonance and the second order term, two-boson (spinon) result, gives the peak at "SPE" energy as near resonance, but it still has relatively small weights as compared to the first order CDE.

Expanding the exponentials, keeping only the one-boson term and integrating explicitly, gives the one-boson transition rate as a delta function at $\nu = qv_\rho$ with

the spectral weight ($\alpha < 1$)

$$W_1 = \frac{2L\Gamma^2(-\alpha)}{qv_\rho^2}\left|\left(\frac{\omega_R-\omega_q}{E_0}\right)^\alpha - \left(\frac{\omega_R+\omega_q}{E_0}\right)^\alpha\right|^2, \quad (10)$$

where $\omega_q \equiv qv_\rho/2$, neglecting v_F^V for simplicity. E_0 is the energy scale depending on the interaction range and roughly of the order of Fermi energy, E_F^c. For $\omega_q \ll |\omega_R|$, $W_1 \propto |\omega_R|^{2\alpha-2}$, while for $\omega_R = 0$, $W_1 \propto \sin^2(\pi\alpha/2)$. Thus LL effects enter the CDE portion (one-boson) of the spectrum in two ways (for short-ranged interaction): first, far from resonance, it changes the frequency dependence of spectral weights from ω_R^{-2}, the noninteracting result, to $\omega_R^{-2+2\alpha}$ (note that all other higher order bosonic contribution decays much faster, this confirms the validity of the bosonic expansion we mentioned above). and secondly as on resonance ($\omega_R = 0$) it changes the value to be nonzero due to finite interaction strength.

To second order, two new effects appear. In the density spectrum, branch mixing process appear. These lead to a continuum absorption beginning at the CDE threshold, $\omega = qv_\rho$. In addition, an $S = 0$ combination of spin excitations may be excited via the two spinon, $\langle\sigma\sigma, \sigma\sigma\rangle$ (note that there is *no* first order contribution in spin channel due to the selection rule of RRS in the polarized spectroscopy), and gives the so-called "SPE" mode at $\nu = qv_F^c$.

In Fig. 1, we show the spectrum from one and two bosons in different resonance energy. One can find that (i) the overall spectral weights decays very fast off resonance, and (ii) the "SPE" peak is generated at $\omega \sim 0.2E_F^c$ by the two-boson contribution near resonance. But as compared with the CDE peak at plasmon energy (about 0.57 E_F^c), the "SPE" peak is still very small compared with CDE. This striking result arises from the fact that the contribution of one spin-boson in the first order is forbidden by the specific selection rule of polarization in depolarized RRS spectroscopy. (iii) At higher energy side above CDE peak, there is some continuum structure which is not shown in the range of Fig. 1. This continuum is from the interaction between different branches of charge bosons due to finite g_2 interaction. We are not interested in their structure because it goes to zero near the plasmon energy and their higher energy behavior is off the experimentally measurable region, and become unphysical due to the failure of the linear dispersion assumption. (iv) When including three or higher order boson contribution (not shown in this paper), we will see the mixture of charge boson and spin boson in a form like $\langle\sigma\sigma\rho, \rho\sigma\sigma\rangle$, which will come into the energy between qv_F^c and qv_ρ, plasmon energy, as a continuum structure. A detailed analysis shows that these higher order contribution is relatively small and no special structure compared to the first two order result we present here. While Fig. 1 is for a specific value of α (= 0.3) we show in Fig. 2 the calculated charge boson and spin boson RRS spectral weights at resonance and away from resonance. In general, the LL theory predicts much smaller spectral weight for the lower energy "SPE" mode than the FL theory [9] at resonance. This is particularly true since our best estimate for the Luttinger exponent of the experimental system [6] (obtained from the CDE energy dispersion) is $\alpha \sim 0.4$.

As compared with the experimental result, which shows possible comparable spectral weight of "SPE" with CDE [6], we find that the LL theory result induced by resonance effect does *not* explain the experimental results quantitatively, even though we could recover the SPE peak through the coupling of two spinon in LL, not as the SPE in FL theory. This inconsistency cannot be resolved even evaluating the full bosonic contribution without expansion as what we have in this paper. Our future work shows that the spectral weight of the "SPE" peak enhanced by spinon coupling in polarized RRS spectra is always relatively small compared to that of CDE. Therefore, unlike the conclusion of previous work [10] based on the incorrect matrix element, we claim that the whole problem cannot be simply understood by the correct LL theory. We believe that the existing experimental results [6] are in the high energy crossover regime where in fact a FL description maybe more appropriate for the RRS data than the LL description which is an asymptotic low energy description. This explains the spectacular quantitative success of the FL RRS theory developed in ref. [9].

In conclusion, we provide the correct LL theory for the RRS spectra calculation, and obtain some meaningful and interesting results to study the possible origin of LL features in the RRS spectra of 1D QWR systems. We also develop an useful bosonic expansion method to study the two-particle correlation function. Finally, we find that the LL theory cannot quantitatively explain the experimental data most likely because the RRS experiments are not in the asymptotic low energy LL regime.

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